Reply to "Comment on 'Correlated noise in a logistic growth model' "

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We respond to the preceding Comment by Behera and O'Rourke, which points out an error in our earlier work [Phys. Rev. E **67**, 022903 (2003)], and argues that our results are not correct. We agree with Behera and O'Rourke: a sign error led us to solve the model for the opposite (positive) sign of the correlation parameter from that which we intended. In order to improve this problem, we extend the range of the correlation parameter λ from (0,1) to (–1,1].

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In the Comment [1] on our paper [2], it is pointed out that our conclusion that large values of the correlation parameter can cause tumor cell extinction is incorrect. The authors of [1] find the reverse behavior—that increasing the correlation parameter promotes the stable growth of tumor cells. However, it is noted that our results are originally based on two positive correlated noises terms. Because of negligence, a sign in Eq. (13) in [2] is not correct, which leads to the different results shown in the Comment. We should point that our results [2] are correct and all figures can be reproduced when the correlation parameter is positive. To correct the mistake and improve our results in our previous paper, we extend the range of λ from (0,1) to $(-1,1]$. When the sign of the noise $\Gamma(t)$ in Eq. (2) in [2] is positive, we can obtain the complete expression for the steady probability distribution function for $-1<\lambda\leq 1$,

$$P_{st}(x)=\begin{cases} NB(x)^{C-1/2}\exp\left(f(x)+\dfrac{E}{\sqrt{D\alpha(1-\lambda^2)}}\arctan\dfrac{Dx+\lambda\sqrt{\alpha D}}{\sqrt{D\alpha(1-\lambda^2)}}\right), & -1<\lambda<1, \\ N'B(x)^{C'-1/2}\exp\left(f(x)+\dfrac{E'}{Dx+\sqrt{D\alpha}}\right), & \lambda=1, \end{cases} \quad (1)$$

where $f(x)=-(b/D)x$, $C=(a+2\lambda\sqrt{(\alpha/D)}b)/2D$, $E=b\alpha/D-(a+2\lambda\sqrt{\alpha/D}b)\lambda\sqrt{\alpha/D}$, $C'=(a+2\sqrt{\alpha/D}b)/2D$, $E'=\sqrt{\alpha/D}(a+b\sqrt{\alpha/D})$, and the other quantities are the same as in Ref. [2].

When the correlation parameter is positive, all results in our previous paper [2] are correct and all figures can be reproduced. The reverse results in the Comment will appear when the correlation parameter is negative. These results can be justified using the expression for the extrema obeyed by the function $P_{st}(x)$. The extrema of $P_{st}(x)$ obey the general equation

$$bx^2+(D-a)x+\lambda\sqrt{D\alpha}=0. \quad (2)$$

Figure 1 shows the solutions of Eq. (2). When the correlation parameter λ is positive, there are two solutions for positive values of x. So there are two extrema, one maximum and one minimum. It is clear that as λ increases the peaks shift toward smaller values of x. This implies that higher values of λ lead to the extinction of cells. Therefore, our previous conclusion [2] that increasing λ may cause cell extinction is correct for positively correlated noise. When the correlation parameter λ is negative, there is only one solution for positive values of x. In this case, the peaks shift toward

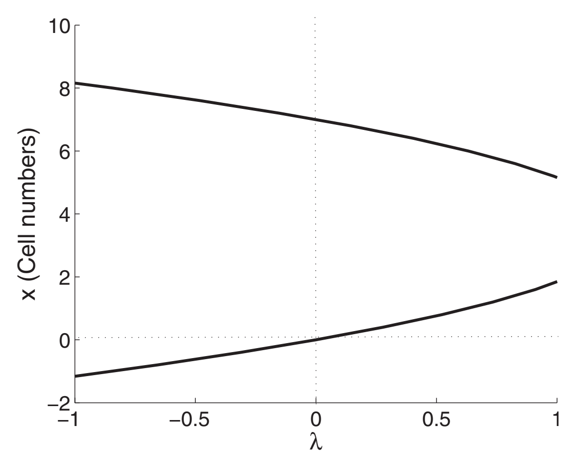

FIG. 1. Plot of extrema of steady-state probability distribution function as a function of λ at $D=0.3$, $\alpha=3.0$, $a=1.0$, and $b=0.1$.

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larger values of x as $|\lambda|$ increases. The higher values of $|\lambda|$ promote cell growth instead of the extinction of cells. Therefore, for negative correlation, the results in the Comment appear.

In this Reply, we pointed out a serious sign error in our earlier work which induces different results from our original paper. In order to correct the mistake and improve the results, we extended the range of the correlation parameter $(-1<\lambda\leq 1)$. It is noted that our previous results [2] are right for positive correlation parameter and all figures can be reproduced, and the results in the Comment appear when the correlation parameter is negative. It is noted that there is an error in the caption of Fig. 3 in our previous paper [2]. The strength α of the additive noise should be 0.5, 3.0, 5.0, 10.0, instead of 0.5, 1.0, 2.0, 3.0. It must be pointed that the cell number in our study is a relative number which denotes the concentration of the cell. When our results are applied to a realistic biological system, the equation's parameters must be determined by the experimental data. When two noises terms have a common origin, we think that they are correlated. However, how to find a proof for all the correlation and memory effects which exist at the microscopic level is still an open problem.

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[1] A. Behera and S. F. O'Rourke, precedings Comment, Phys. Rev. E **77**, 013901 (2008).

[2] B.-Q. Ai, X. J. Wang, G. T. Liu, and L. G. Liu, Phys. Rev. E **67**, 022903 (2003).